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ASSET RISK AND LEVERAGE UNDER INFORMATION ASYMMETRY

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Asset risk and leverage under information asymmetry

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Purpose – The aim of this paper is to investigate whether higher asset risk can be associated with higher leverage and to provide a rationale for the relatively low debt ratios displayed by many firms.

Design/methodology/approach – We model a game between an informed firm and uninformed lenders. The firm knows the risk of its assets, while lenders only know the minimum level of risk. We solve for the signaling equilibrium and derive explicit formulas for the firm's cost of debt and optimal leverage.

Findings – In contrast to the tradeoff theory of capital structure, we find that asset risk and leverage are positively (instead of negatively) related. Furthermore, leverage is lower than when lenders are informed about the firm's risk. We illustrate these results with a numerical example.

Practical implications – Low-risk firms can choose a lower leverage to signal their lower risk and reduce their cost of debt. High-risk firms may prefer to pay higher interest rates and use higher leverage.

Originality/value – The paper is able to explain why some firms use surprisingly low leverage and do not appear to take advantage of their debt tax shields.

Keywords: capital structure, leverage, asset risk, cost of debt, information asymmetry

1. Introduction

The tradeoff theory of capital structure implies a negative association between a firm's financial leverage and the riskiness of its assets. Bradley, Jarrell, and Kim (1984) argue that asset risk increases the present value of leverage-related costs (i.e., bankruptcy costs, agency costs of debt, and loss of non-debt tax shields); thus decreasing the firm's optimal debt level. Castanias (1983) provides evidence of a negative relation between bankruptcy risk and several leverage indicators. Furthermore, Bessler, Drobetz, Haller and Meier (2013) note that the increased proportion of zero-leverage firms in the economy can be explained by an increase in asset risk. In contrast, Kale, Noe and Ramirez (1991) argue that, when personal taxes are considered, the relation can become positive at higher levels of risk. Claessens, Djankov and Nenova (2005) substantiate this positive relation in a cross-country analysis. More recently, Choi and Richardson (2016) report a negative relation between asset risk and leverage. However, half of the firms exhibit a large dispersion in leverage, while their asset risk is similar. For the other half, leverage is similar, but asset risk varies considerably across firms.

In the area of asset pricing, structural debt models determine corporate debt prices using asset risk (volatility) and financial leverage (capital structure) as basic input variables. A typical example is Merton (1974). Bohn (2000) provides a survey of such models. Corporate debt needs to provide a risk premium over the risk-free rate to offset the potential loss associated with the firm's default on its payment obligations. In these models, asset risk is known to investors (lenders) and no particular constraint is imposed on the relation between asset risk and leverage.

In this paper, we investigate the case where only the firm knows (is informed about) its asset risk. Lenders are uninformed, but can observe the firm's leverage (capital structure) choice to infer its true asset risk. Conversely, the firm can use its leverage to signal that its asset risk is low and therefore decrease its cost of debt (and cost of capital). The central feature of the model is that higher asset risk is associated with higher leverage. In other words, financial leverage and asset risk are complementary rather than substitute decision variables. The model thus provides a rationale for the empirical findings of Claessens, Djankov and Nenova (2005). It also yields an explicit formula that complements the marginal analysis of Kale, Noe and Ramirez (1991). Another appealing feature of the model lies in its ability to explain why some low-risk firms adopt conservative debt ratios, instead of taking greater advantage of the debt tax shield. Molina (2005) emphasizes the same puzzle, but argues instead that distress costs have been underestimated by ignoring the endogeneity of leverage.

The rest of the paper is organized as follows. Section 2 presents the model's assumptions. Section 3 provides an analytical solution for optimal leverage (capital structure) and debt pricing. Section 4 compares the solutions under perfect and imperfect information. Section 5 concludes.

2. Modelling assumptions

The model involves a game between an informed firm and uninformed lenders. The firm is endowed with equity capital, which is set to 1 without loss of generality. To supplement its equity capital, the firm can borrow $w - 1$ from lenders. Hence, the firm's balance sheet size is w. Abusing notations, I will subsequently refer to w as leverage. The firm's resources are invested in a risky asset, whose returns depend on the states of nature. Uncertainty is represented by the existence of two states in the end period. Each state may occur with the same probability 0.5. Asset returns are assumed to follow a simple binomial process defined by

$$\begin{cases} \mu + \sigma & : \text{in the good state} \\ \mu - \sigma & : \text{in the bad state} \end{cases}$$

The firm is informed about the asset's expected return μ and risk σ. Lenders know μ and the minimum level of risk σ^*, but are uninformed about the firm's asset risk σ. Since asset risk is private information, the interest rate $k_d(.)$ that lenders charge cannot depend on the firm's asset risk, but must be defined as a function of the observed firm leverage *w*. For a given leverage *w*, and in the absence of default, the gross return to the firm's shareholders is

$$\begin{cases} w(\mu + \sigma) + (1 - w)k_d & : \text{in the good state} \\ w(\mu - \sigma) + (1 - w)k_d & : \text{in the bad nstate} \end{cases}$$

The firm's shareholders are assumed to have a low degree of risk aversion. This assumption entails that leverage is large, which ensures that $w(\mu - \sigma) + (1 - w)k_d < 0$ in the bad state. However, because of limited liability, the firm can default on its debt payment $(w - 1)k_d$ in the bad state. In that event, ownership of the firm's assets is transferred to lenders. Hence, the gross return to lenders (i.e., including repayment of the principal) can be specified as:

$$\begin{cases} k_d & : \text{in the good state} \\ (\mu - \sigma)\dfrac{w}{w - 1} & : \text{in the bad state} \end{cases}$$

The firm's objective is to select the leverage *w* that maximizes an expected utility function *U*(.) conditional on the cost of debt $k_d(.)$. The utility function is assumed to verify the usual conditions (increasing marginal utility and concavity). The objective of the firm's lenders is to select the interest rate schedule $k_d(.)$ that maximizes their expected return. However, competition in the debt market implies that their expected return is actually the risk-free rate R_f.

3. Equilibrium leverage and cost of debt

Given that the firm defaults in the bad state, the optimal leverage maximizes the expected utility

$$EU(.) = \frac{1}{2}U(w(\mu+\sigma)+(1-w)k_d) + \frac{1}{2}U(0)$$

The first order condition is therefore:

$$\frac{dEU(.)}{dw} = \frac{1}{2}U'(.)\times\frac{d}{dw}\left[w(\mu+\sigma)+(1-w)k_d\right] = 0$$

Since $U'(.) > 0$, the first order condition reduces to the following

$$(\mu+\sigma-k_d)+(1-w)k'_d = 0 \tag{1}$$

The above equation indicates that, at the optimum leverage, the additional return from increasing the firm's size (first term on the left hand side) is offset by an increase in borrowing costs (second term on the left hand side).

The equilibrium condition that lenders achieve on average the risk-free rate leads to the equation

$$k_d(w)+(\mu-\sigma)\frac{w}{w-1} = 2R_f \tag{2}$$

In equilibrium, the lending rate is determined as if lenders know the firm's true asset risk σ. As a result, σ can be extracted from equation (2) to give

$$\sigma = \mu + (k_d - 2R_f)\frac{w-1}{w}$$

Substituting this expression into the first order condition (1) yields a first order linear differential equation in $k_d(.)$ that only depends on the observed firm leverage *w*:

$$2\mu - 2R_f\frac{w-1}{w} - \frac{1}{w}k_d + (1-w)k'_d = 0 \qquad (3)$$

This equation can be integrated to provide the firm's cost of debt:[1]

$$k_d(w) = \frac{w}{w-1}\left[C - \frac{2R_f}{w} + 2(\mu - R_f)\log w\right] \qquad (4)$$

To determine the constant of integration *C*, first substitute expression (4) into the equilibrium condition (2). After simplification, the firm's optimal leverage *w* is given by:

$$w(\sigma) = \exp\left\{\frac{2R_f - \mu + \sigma - C}{2(\mu - R_f)}\right\} \qquad (5)$$

To simplify subsequent expressions, it is convenient to use the asset risk premium $\pi \equiv \mu - R_f$ and drop the expected asset return μ. Equations (4) and (5) can then be expressed as:

[1] Checking that the functional form in equation 4 satisfies the differential equation 3 is straightforward.

$$k_d(w) = \frac{w}{w-1}\left[C - \frac{2R_f}{w} + 2\pi \log w\right] \tag{4'}$$

$$w(\sigma) = \exp\left\{\frac{R_f + \sigma - \pi - C}{2\pi}\right\} \tag{5'}$$

It is important to recognize from equation (5') that leverage $w(.)$ is an *increasing* function of asset risk σ. In equilibrium, the firm can thus signal its lower risk by selecting a lower leverage.

Now, consider the firm with the minimum asset risk σ^*. Let w^* denote the maximum leverage beyond which the probability of defaulting is nonzero. This means that upon choosing w^* the asset payoff in the bad state is just able to cover the debt repayment:

$$w^*(\mu - \sigma^*) + (1 - w^*)k_d(w^*) = 0. \tag{6}$$

Then, because default cannot occur, the firm's debt is riskless. This implies that lenders must receive the risk-free rate: $k_d(w*) = R_f$. Substituting this expression into equation (6) provides:

$$w^*(\mu - \sigma^*) + (1 - w^*)R_f = 0$$

Simple application of algebra then yields:

$$w^* = \frac{R_f}{R_f - \mu + \sigma^*} = \frac{R_f}{\sigma^* - \pi} \tag{7}$$

Upon observing the firm's leverage w^*, lenders can infer that the firm's asset risk is σ^*. Indeed, suppose that the risk is higher: $\sigma > \sigma^*$. The fact that the optimal signaling strategy is *increasing* in σ implies that the leverage for the firm with asset risk σ^* will be *lower* than w^*. However, this firm would not be optimally leveraged, given that it will not default. Hence, it can be concluded that the optimal leverage function satisfies the condition $w(\sigma^*) = w^*$. Since debt is risk-free, if observed leverage is w^*, then $k_d(w^*) = R_f$.

This equation combined with formula (4') evaluated at w^* yields:

$$\frac{R_f}{R_f - \sigma^* + \pi}\left[C - 2R_f \frac{\sigma^* - \pi}{R_f} + 2\pi \log \frac{R_f}{\sigma^* - \pi}\right] = R_f$$

Solving for the constant gives:

$$C = R_f + \sigma^* - \pi - 2\pi \log \frac{R_f}{\sigma^* - \pi}$$

This expression can then be substituted back into equations (4') and (5') to provide the equilibrium cost of debt and firm leverage. The following proposition summarizes the analytical results.

Proposition 1: Under the assumptions stated in section 2, the firm's cost of debt is

$$k_d(w) = \frac{w}{w-1}\left[R_f + \sigma^* - \pi - \frac{2R_f}{w} + 2\pi \log \frac{w}{w^*}\right] \tag{8}$$

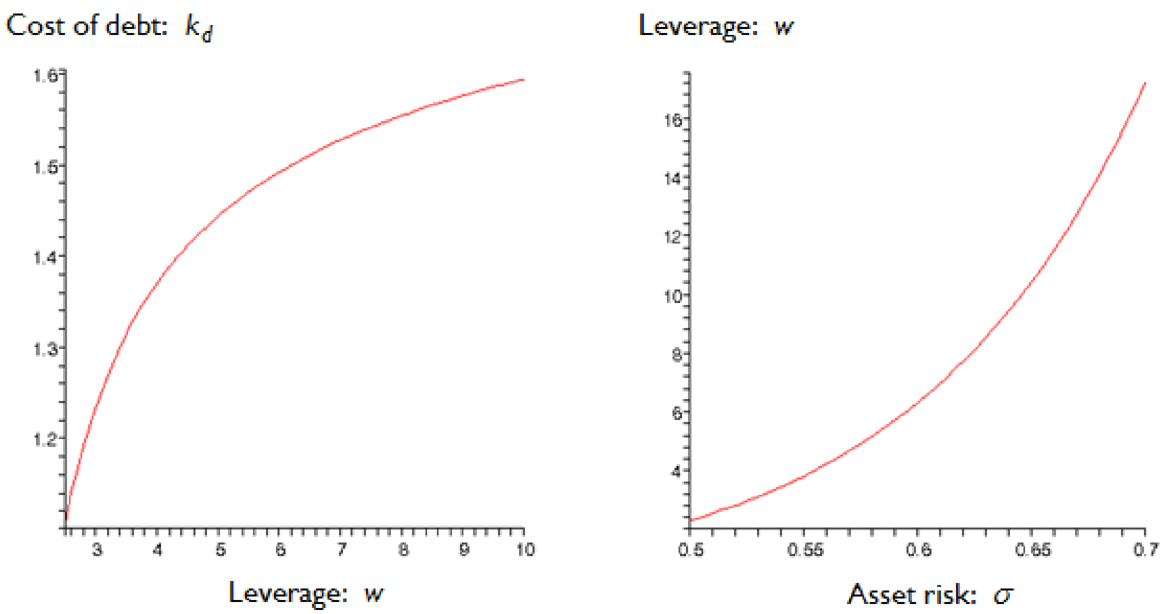
with w^* given in equation (7). The firm's optimal leverage for asset risk σ is

$$w(\sigma) = w^* \exp\left\{\frac{\sigma - \sigma^*}{2\pi}\right\} \tag{9}$$

Figure 1 illustrates the equilibrium leverage and cost of debt for the following parameters: $R_f = 1.05$; $\mu = 1.10$; and $\sigma^* = 0.5$. Note that the risk-free rate and the risky asset's expected return are inclusive of principal repayment.

Fig. 1

The left panel plots the firm's cost of debt conditional on its selected leverage; the right panel plots the firm's optimum leverage as a function of its asset risk.



Equation (8) indicates that the cost of debt is an increasing function of leverage. At constant asset risk, higher leverage is associated with greater loss in the bad state. Accordingly, higher leverage must be compensated by higher cost of debt (interest rate). In addition, the model implies that higher leverage is associated with higher asset risk, which requires a higher cost of debt at constant leverage. In structural models of corporate debt, such as Merton (1974), firms with higher leverage can display a lower a cost of debt provided that their asset risk is lower.

Equation (9) reveals that leverage is increasing with risk. This result is consistent with the recent cross-country evidence reported in Claessens, Djankov and Nenova (2005). On the other hand, the result contradicts the tradeoff theory of capital structure, in which asset risk and leverage are negatively correlated. For instance, Castanias (1983) and Bradley, Jarrell, and Kim (1984) report a negative relationship between risk and leverage. Equation (9) also indicates that leverage is a convex function of asset risk, consistent with the empirical evidence presented by Kale, Noe and Ramirez (1991).

In addition, it can be verified that, with leverage defined through equation (9), the cost of debt given in equation (8) becomes:

$$k_d(\sigma(w)) = 2R_f - \frac{w}{w-1}(\mu - \sigma) \tag{10}$$

Equation (10) indicates that lenders are compensated (or set interest rates) as under perfect information. As usual, the equilibrium condition requires that lenders (as principals) do not bear the cost of information asymmetry.

4. Comparison with the case of perfect information

Under information asymmetry, the firm has to bear the cost of signaling its private (asset risk) information. To illustrate this point, consider the firm with minimum risk σ^*. If lenders are informed about asset risk, the firm can increase its leverage to any arbitrarily high level $w > w^*$. For a given leverage *w*, lenders must charge the following interest rate, including principal, which ensures that they receive on average the risk-free rate:

$$\hat{k}_d(w) = 2R_f - \frac{w}{w-1}(\mu - \sigma^*) \tag{11}$$

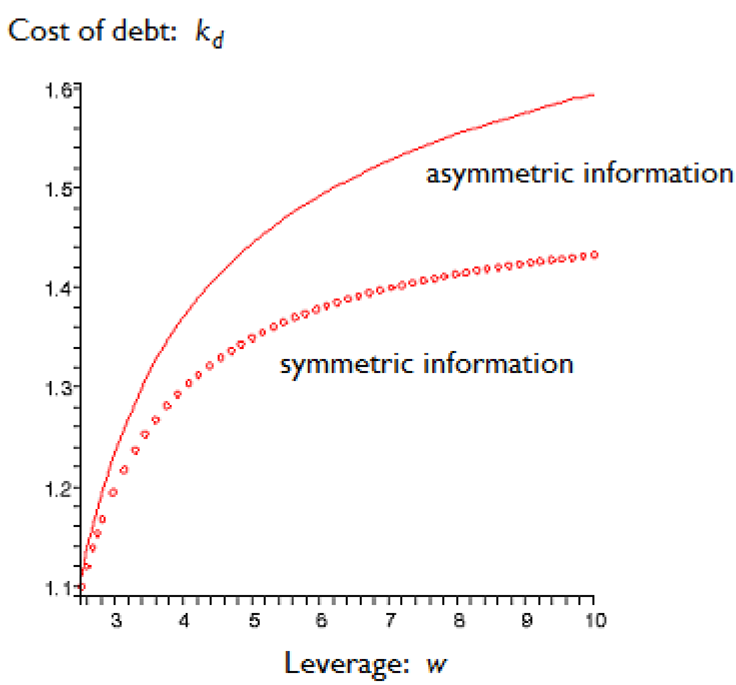
However, under information asymmetry, lenders will set the following interest rate (cost of debt) upon observing the firm's leverage *w*:

$$k_d(w) = \frac{w}{w-1}\left[R_f + \sigma^* - \pi - \frac{2R_f}{w} + 2\pi \log\frac{w}{w^*}\right] \tag{12}$$

Figure 2 illustrates the firm's cost of debt under symmetric and asymmetric information using the same parameters as in Figure 1 (i.e. $R_f = 1.05$; $\mu = 1.10$; and $\sigma^* = 0.5$).

Fig. 2

The figure plots the firm's cost of debt when lenders are informed and uninformed about the firm's asset risk.



The cost of debt in (12) is higher relative to the cost of debt in (11) for the reason that lenders take into account the increasing asset risk associated with higher leverage. Hence, the higher cost of debt constrains the firm to adopt a lower leverage relative to the optimal level achieved under symmetric information. Information asymmetry may explain why firms adopt conservative debt ratios that are inconsistent with levels predicted by the standard tradeoff theory of capital structure. Strebulaev and Yang (2013) point out that these firms leave considerable amount of money on the table by not levering up. Their explanation is that under-diversified wealth causes management preferences to differ from those of shareholders.

Information asymmetry thus entails a welfare loss for the firm's shareholders, which is distinct from the moral hazard argument proposed in Strebulaev and Yang (2013). To measure this loss, suppose, for instance, that the optimal leverage under symmetric information is w^{**}. The firm's return is zero in the bad state. With leverage w and the cost of debt given by equation (11), the firm's return in the good state is given by

$$
\begin{aligned}
w(\mu-\sigma^{*})+(1-w)k_{d} &= w(\mu-\sigma^{*})+(1-w)\left[2R_{f}-\frac{w}{w-1}(\mu-\sigma^{*})\right] \\
&= 2R_{f}+2w\pi
\end{aligned}
$$

By contrast, if the cost of debt follows equation (12), the firm must decrease its leverage to $w^{*}<w^{**}$. As a result, the loss in expected utility is

$$
\frac{1}{2}U\left(2R_{f}+2w^{*}\pi\right)-\frac{1}{2}U\left(2R_{f}+2w^{**}\pi\right)<0
$$

which is negative, since the utility function is increasing.

5. Conclusion

Under asymmetric information, higher asset risk can be associated with higher leverage. This result is opposite to the standard prediction of the tradeoff theory of capital structure, which assumes that lenders and borrowers have the same information. However, it is consistent with empirical findings presented by Claessens, Djankov and Nenova (2005). The reason for this positive relation is that lower leverage is used for signaling a lower asset risk and reducing the firm's cost of debt. As a result, leverage is lower than under perfect information. This paper provides a rationale for the conservative debt ratios of low risk firms, which has been a longstanding puzzle that Molina (2005) puts down to statistical measurement problems arising from the endogeneity of leverage. In this paper, leverage is endogenously constrained to induce a higher sensitivity to the firm's asset risk. We thus offer an alternative explanation to the underleverage phenomenon that differs from those based on management risk aversion and moral hazard (Strebulaev and Yang, 2013).

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